

M 2/19

Exam I Rework
(Due W 2/21)

Important special case $Y = aX + b, a \neq 0$

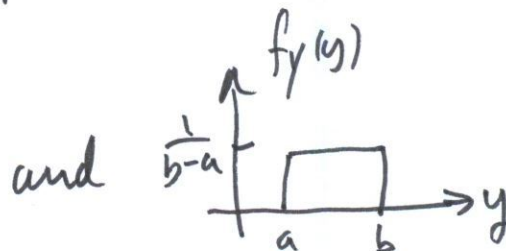
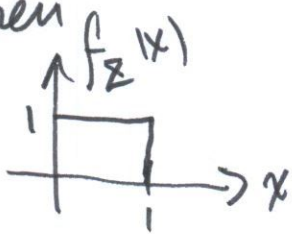
Then $f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y-b}{a}\right)$ \leftarrow stretched, shifted, and scaled version of f_X .

Examples 1.) $X \sim \text{unif}(0,1)$

(possibly flipped - if $a < 0$)

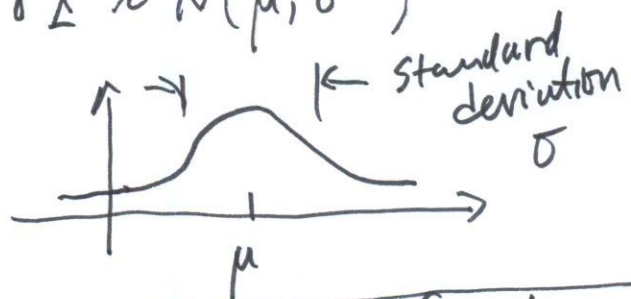
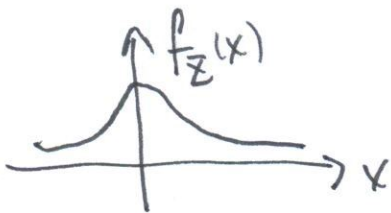
Let $a < b$. If $Y = a + (b-a)X$.

Then



$Y \sim \text{unif}(a,b)$

2.) $X \sim N(0,1)$ $Y = \mu + \sigma X \sim N(\mu, \sigma^2)$

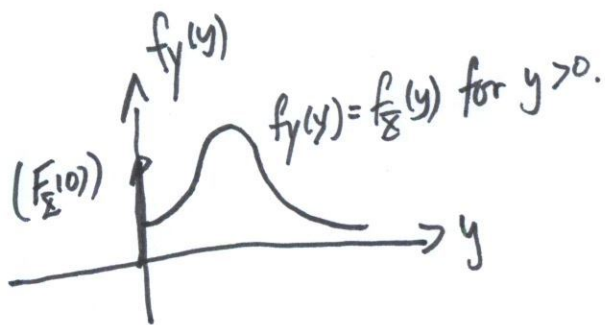
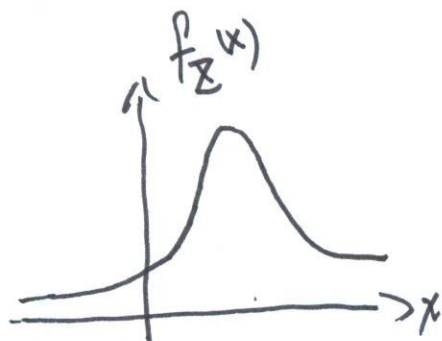
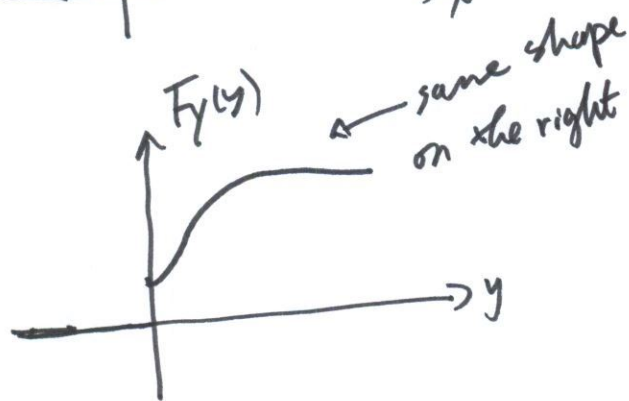
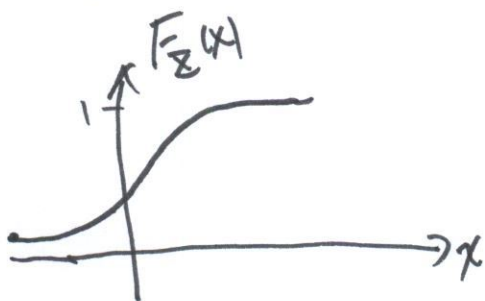
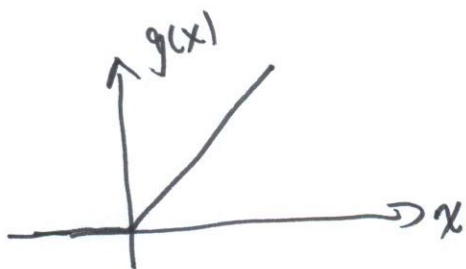


In engineering, we sometimes deal with functions that do not satisfy the conditions for the

fundamental theorem.

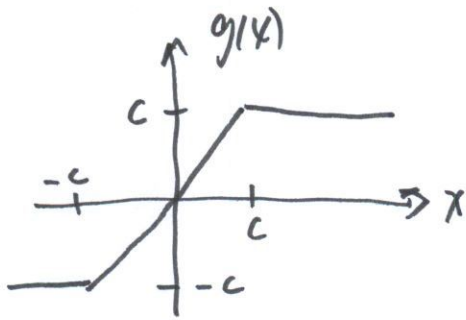
Examples Some Standard Engineering Operations

1. Rectifier $g(x) = \begin{cases} x & x \geq 0 \\ 0 & \text{o.w.} \end{cases}$

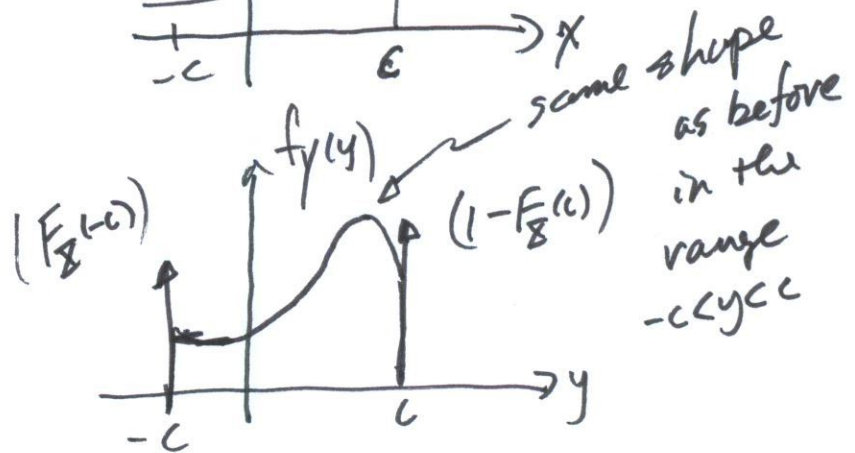
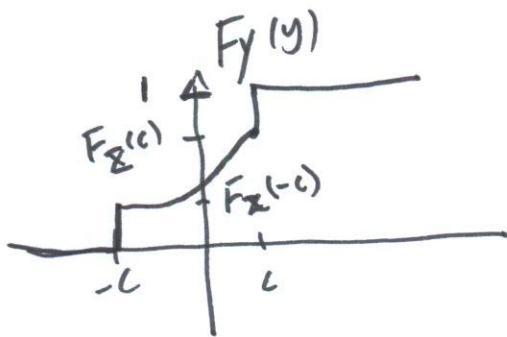
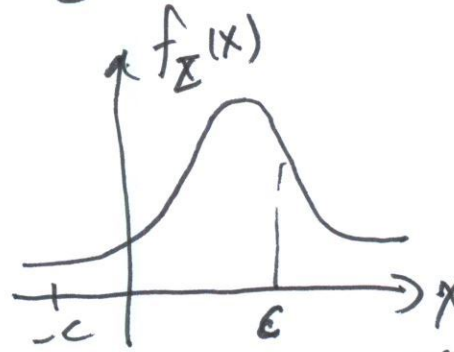
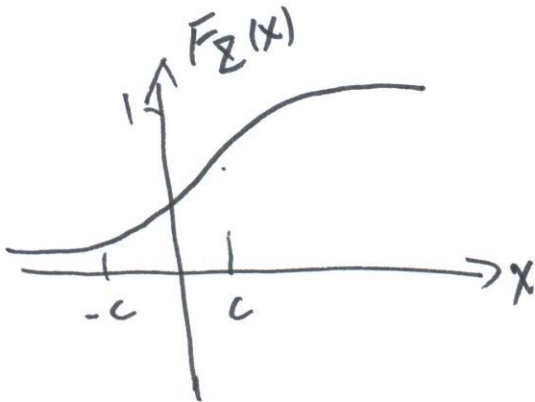


$$f_Y(y) = F_Z(0) \delta(y) + f_Z(y) u_3(y)$$

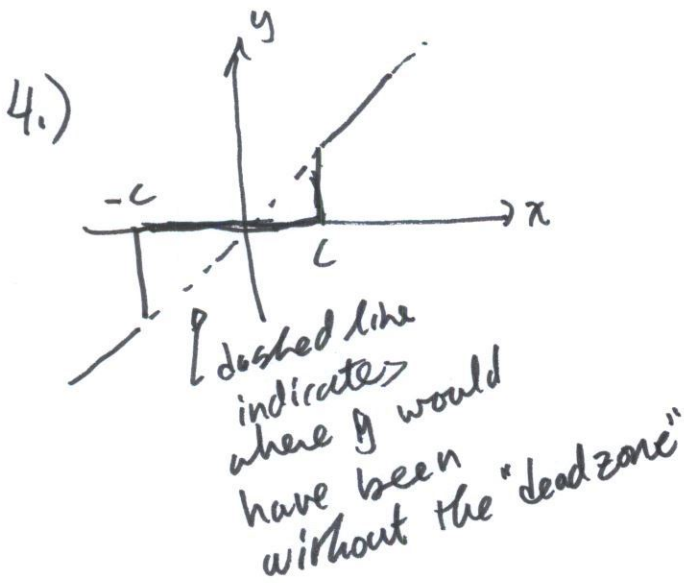
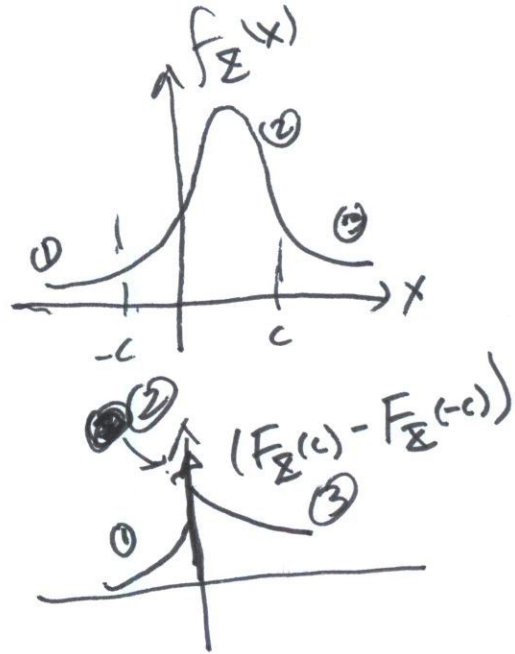
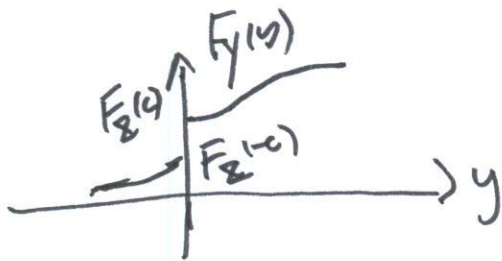
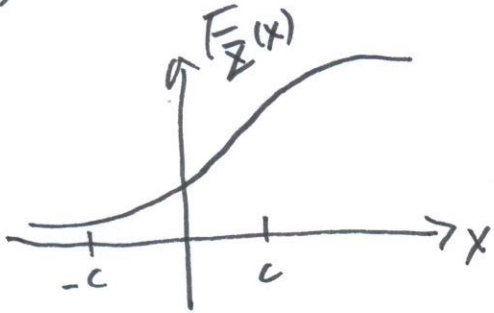
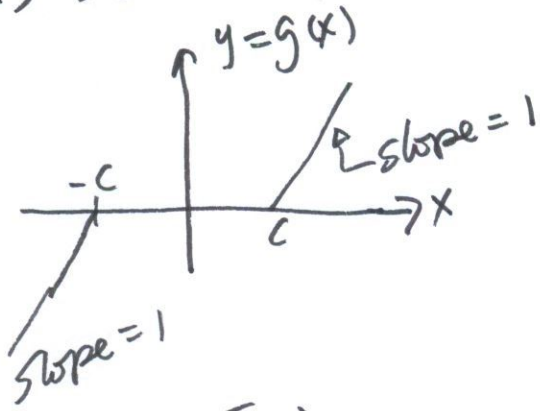
2.) Saturation

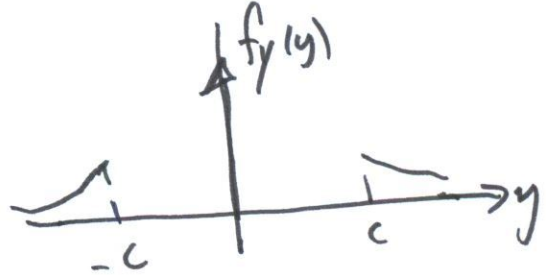
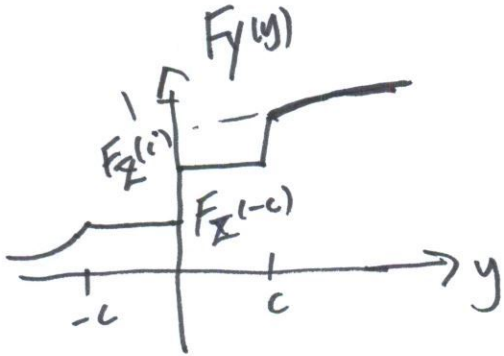
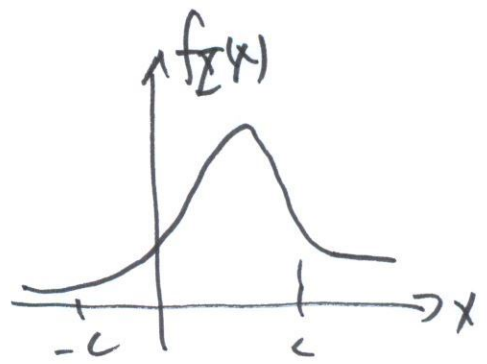
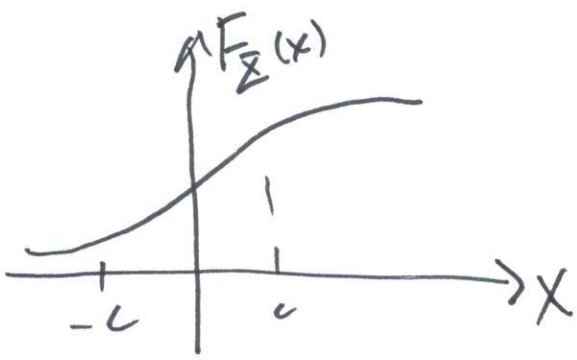


$$g(x) = \begin{cases} -c & x < -c \\ x & -c \leq x \leq c \\ c & x > c \end{cases}$$

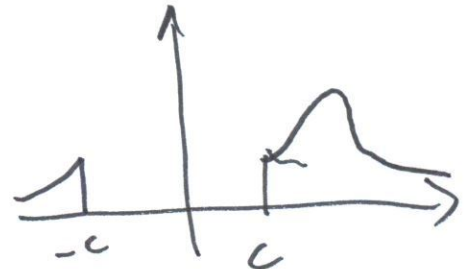
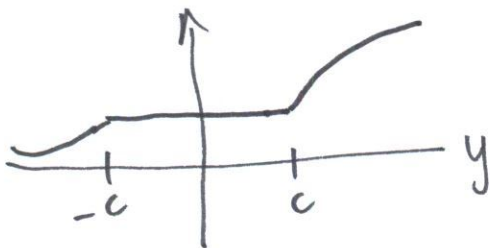
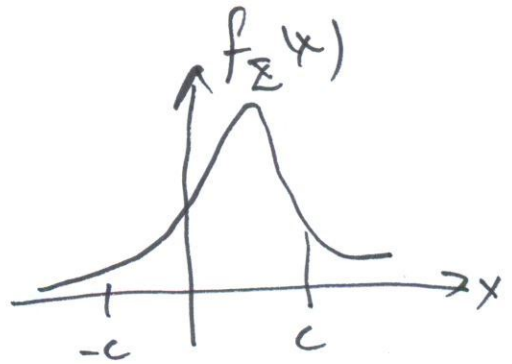
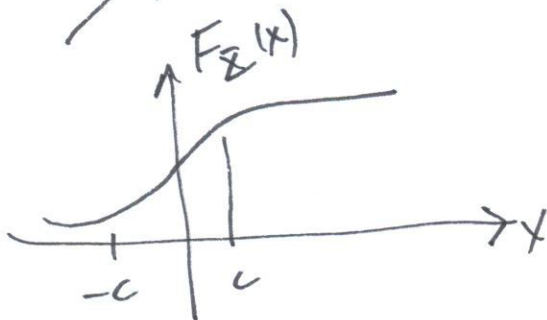
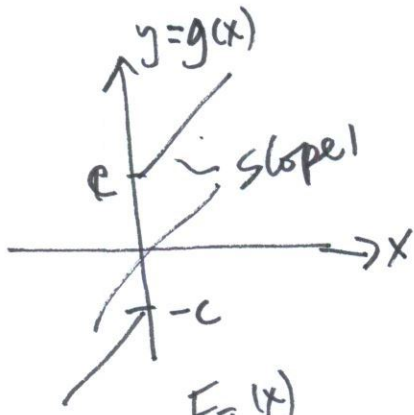


3.) Dead zone





5.)



6.) Quantizer



See Fig 3.2-11 (p. 164)

Functions of the Form $Z = g(X, Y)$

Important Special Cases

1.) Rayleigh Distribution $Z = \sqrt{X^2 + Y^2}$
where X, Y are iid $N(0, \sigma^2)$ RV's.

$$F_Z(z) = P(Z \leq z) = P(\sqrt{X^2 + Y^2} \leq z)$$

$$= \iint_{x^2 + y^2 \leq z^2} \frac{1}{2\pi} e^{-\frac{x^2 + y^2}{2\sigma^2}} dx dy$$

$$= \int_0^{2\pi} \int_0^z \frac{1}{2\pi} e^{-\frac{r^2}{2\sigma^2}} r dr d\theta$$

$$= \int_0^z r e^{-\frac{r^2}{2\sigma^2}} dr = -e^{-\frac{r^2}{2\sigma^2}} \Big|_0^z$$

$$F_z(z) = \left[1 - e^{-\frac{z^2}{2\sigma^2}}\right] u_5(z)$$

$$f_z(z) = \frac{d}{dz} [F_z(z)] = \frac{z}{\sigma^2} e^{-\frac{z^2}{2\sigma^2}} u_5(z)$$

