EEE5542 Random Processes - Spring 2018

Requirements: Graduate Standing

Instructor: Dr. Rodney Roberts  
Room A360  
(850) 410-6458  
rroberts@eng.fsu.edu

Schedule: Monday, Wednesday 11:00 am - 12:15 pm, Room A317

Office Hours: Monday 1:30 - 2:30 pm or by appointment


Goals: Random processes; analysis and processing of random signals; modeling of engineering systems by random processes, selected applications in detection; filtering; reliability analysis; and system performance modeling.

Course Outline:

1. Elementary probability - sets, sample spaces, axioms, joint and conditional probability, Bayes' Theorem
2. Random variables - concepts, distribution and density functions
3. Operations on random variables - expectation, moments, transformation of random variables
4. Multiple random variables
5. Random sequences
6. Random processes
7. Linear Systems with random inputs

Grading Policy:

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<tr>
<th>Grade</th>
<th>Percentage</th>
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<tr>
<td>Exam 1</td>
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<td>Exam 2</td>
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<td>Exam 3</td>
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<td>HW/Quizzes</td>
<td>10%</td>
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<td>Final Exam</td>
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Homework: Late homework will not be accepted.

Honor Code: Students are bound by the Academic Honor Code of their university.

Policy Statements

- Class attendance is mandatory.
- Students are bound by the honor code of their university.
- Students with disabilities who need academic accommodations should register with and provide documentation to the University and bring a letter to the instructor indicating the need for accommodations and the specific type.
- If you have to miss an exam, you must notify the instructor prior to the exam. In this case, the Final Exam will be worth that much more.
History/Applications of Probability and Statistics

- Gambling
- Taxes (Statistics - comes from the word state (government))
- Insurance (average - French term for how much money a ship owner should pay to cover possible loss)
- Medical policy (e.g., vaccines)
- Biology / genetics
- Politics (Gallup polls)
- Economics (modeling stock prices by random processes)
- Engineering (communication systems, statistical signal processing, manufacturing, ...)

- Many other applications such as computational intelligence, computational linguistics, 
et al in... letter frequency 
  the, be, to, of, and, ... word frequency
Different Types of Probability

1.) Intuitive probability (Koopman)

2.) Classical Prob. Theory (Equally Likely Outcomes)
   
   \[ S = \text{sample space} = \text{collection of possible outcomes.} \]

   An event is a subset of outcomes.

   The probability of an event \( A \)
   
   \[ P(A) = \frac{\#(A)}{\#(S)} \]

Founders of Classical Prob. Theory

- Cardano, Galileo
- Fermat, Pascal, Huygens
- Bernoulli
- Laplace

Example: Roll a pair of fair dice

\[
\begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 & 6 \\
1 & 2 & 3 & 4 & 5 & 6 \\
2 & 3 & 4 & 5 & 6 & 7 \\
3 & 4 & 5 & 6 & 7 & 8 \\
4 & 5 & 6 & 7 & 8 & 9 \\
5 & 6 & 7 & 8 & 9 & 10 \\
6 & 7 & 8 & 9 & 10 & 11 \\
\end{array}
\]

\[ P(S=2) = \frac{1}{36} \]

\[ P(S=3) = \frac{2}{36} = \frac{1}{18} \]

\[ P(S=4) = \frac{3}{36} = \frac{1}{12} \]
\[ S' = \{ (1,1), (1,2), \ldots, (1,6), \ldots, (6,1), (6,2), \ldots, (6,6) \} \]

\[ \#(S') = 36 \]

3. Relative Frequency Approach (Richard von Mises)

\[ P(A) = \lim_{n \to \infty} \frac{n_A}{n} \]

4. Axiomatic (Modern) Prob. Theory (Andrei Kolmogorov)

(A1) \( P(A) \geq 0 \)

(A2) \( P(S) = 1 \)

(A3) If the events \( A_1, A_2, A_3, \ldots \) are mutually exclusive, then

\[ P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i) \]
Set Theory

\( S = \) universal set = sample space
\( \phi = \) empty set
\( a \in A \) "a is in the set A"
\( a \notin A \) "a is not in the set A"

\( A \cup B, A \cap B, A \setminus B, A \triangleq B \)

\( A^c = "A \text{ complement}" = \{x \in S \mid x \notin A\} \)

\( (A^c)^c = A \)

commutative law: \( A \cup B = B \cup A, A \cap B = B \cap A \)

associative law: \( A \cup (B \cup C) = (A \cup B) \cup C, A \cap (B \cap C) = (A \cap B) \cap C \)

distributive law:

\( (A \cup B) \cap C = (A \cap C) \cup (B \cap C) \)

\( (A \cap B) \cup C = (A \cup C) \cap (B \cup C) \)

De Morgan's Law

\( (A \cup B)^c = A^c \cap B^c \)

\( (A \cap B)^c = A^c \cup B^c \)

Difference \( A - B = \{x \in A \mid x \notin B\} = A \cap B^c \)

Symmetric Difference \( A \triangle B = (A - B) \cup (B - A) \) XOR
Venn Diagram

Sigma Algebra

Consider a universal set $\mathcal{S}$ and let $\mathcal{M}$ be a collection of subsets of $\mathcal{S}$. We say that $\mathcal{M}$ forms an algebra if

1. $\emptyset \in \mathcal{M}$, $\mathcal{S} \in \mathcal{M}$
2. If $E \in \mathcal{M}$ and $F \in \mathcal{M}$ then $E \cup F \in \mathcal{M}$ and $E \cap F \in \mathcal{M}$.
3. If $E \in \mathcal{M}$ then $E^c \in \mathcal{M}$.

$\mathcal{M}$ is a $\sigma$-algebra if Axiom 2 is replaced with

2'. If $E_1, E_2, \ldots \in \mathcal{M}$ then $\bigcup_{i=1}^{\infty} E_i \in \mathcal{M}$ and $\bigcap_{i=1}^{\infty} E_i \in \mathcal{M}$.
Result If \( A \subseteq B \) then \( \#(A) \leq \#(B) \)

Proof \( B = A \cup (B-A) \) is a disjoint union.

\[
\#(B) = \#(A \cup (B-A)) = \#(A) + \#(B-A) \geq \#(A).
\]

Result (Principle of Inclusion and Exclusion - PIE)

\[
\#(A \cup B) = \#(A) + \#(B) - \#(A \cap B)
\]

Proof \( A \cup B = (A-B) \cup (A \cap B) \cup (B-A) \) is a disjoint union.

\[
\Rightarrow \#(A \cup B) = \#(A-B) + \#(A \cap B) + \#(B-A)
\]

Now \( A = (A \cap B^C) \cup (A \cap B) = (A-B) \cup (A \cap B) \) is a disjoint union.

\[
\Rightarrow \#(A) = \#(A-B) + \#(A \cap B) \Rightarrow \#(A-B) = \#(A) - \#(A \cap B)
\]

Similarly, \( \#(B-A) = \#(B) - \#(A \cap B) \)

\[
\Rightarrow \#(A \cup B) = \#(A) - \#(A \cap B) + \#(A \cap B) + \#(B) - \#(A \cap B)
\]

So \( \#(A \cup B) = \#(A) + \#(B) - \#(A \cap B) \).